DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

A Note to Solutions of Problem No. I. by J. H. DRUMMOND, LL. D., Portland, Maine.

I assumed, as I perceive erroneously, that solutions in *integral* numbers were required; hence my remarks that "there are *comparatively* few square numbers" which can be divided into two squares.

Mr. Adcock's solution is a *very fine* one, but it seems to me that it can be put into simpler and much more easily remembered forms, thus $(p^2+q^2)^2=(p^2-q^2)^2+(2pq)^2$. Hence $1=\left(\frac{p^2-q^2}{p^2+q^2}\right)^2+\left(\frac{2pq}{p^2+q^2}\right)^2$ and $n^2=n^2\left(\frac{p^2-q^2}{p^2+q^2}\right)^2+n^2\left(\frac{2pq}{p^2+q^2}\right)^2$, in which p and q may be any unequal numbers; to obtain prime numbers, however, p and q must be prime to each other and one odd and the other even.

2. Proposed by J. M. COLAW, Principal of High School, Monterey, Virginia.

Find two numbers, such that the difference of their squares may be a cube, and the difference of their cubes a square.

III. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let ax^3+b and ax^3-b denote the numbers. The difference of their squares is $4abx^3$, which must be a cube, $=8a^3x$ say, then $b=2a^2$. The difference of their cubes is $6a^2bx^6+2b^3$, which must be a square; or, substituting $2a^2$ for b and then striking out the square factor $4a^4$, $3x^6+4a^2=\square=(2x^3-a)^2$ suppose; whence $x^3=8a$. Take $a=1$, then $x=2$, $b=2$; and the numbers are 10 and 6.

10. Proposed by L. B. HAYWARD, Bingham, Ohio.

Find two numbers such that each of them, and also their sum and their difference, when increased by unity shall all be squares.

III. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let x^2-1 and y^2-1 denote the numbers required and the first two conditions are satisfied in the notation.

But we still have $x^2+y^2-1=\square=v^2$, and $x^2-y^2+1=\square=w^2$, to dispose of. From the first of these we get $y^2=v^2-x^2+1$, and by adding the first and second we get $2x^2=v^2+w^2$.

Let $u=t+u$, $w=t-u$ and the last equation becomes $x^2=t^2+u^2$, which is satisfied by $t=n(p^2-q^2)$, $u=2npq$, and then $x=n(p^2+q^2)$, $v=n(p^2+2pq-q^2)$.

Substituting these values of x and v in $y^2=v^2-x^2+1$ we get

$y^2=4npq(p^2-q^2)+1=\square=(2mn-1)^2$ say; whence $n=\dfrac{m}{m^2-pq(p^2-q^2)}$, where m, p, q may be chosen at pleasure.

If $p=3$, $q=2$, then $n=\frac{m}{m^2-30}$, $=1$ when $m=6$; $\therefore x=13$, $y=11$, and the numbers are 168 and 120.

If $p=4$, $q=1$, then $n=\frac{m}{m^2-60}$, $=2$ whem $m=8$; and then $x=34$, $y=31$, and the numbers are 1155 and 960.

PROBLEMS.

16. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

Find three numbers such that the cube of any one plus the sum of the squares of the other two will be a square.

17. Proposed by ARTEMAS MARTIN, LL. D., U. S Coast and Geodetic Survey Office, Washington, D. C.

Is it possible to find two positive whole numbers such that each of them, and also their sum and their difference, when *diminished* by unity shall all be squares?

Solutions to these problems should be received on or before December 1st.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

8. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Prove that the mean area of all triangles having their vertices upon the surface of a given triangle and bases parallel to the base of the given triangle, is $\frac{13}{210}$ (area of given triangle).

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent AD by a, BC by b, and the area of $\triangle ABC, =\frac{1}{2}ab$, by $\triangle$.